Special Practice Problems sudhir jainam

JEE (Mains & Advanced)

Topics: Basic Maths & Logarithm

• Objective Questions Type I [Only one correct answer] .

In each of the questions below, four choices are given of which only one is correct. You have to select the correct answer which is the most appropriate.

- 1. If $\log_2 3 = a$, $\log_3 5 = b$ and $\log_7 2 = c$, then the logarithm of the number 63 to base 140 is
- (c) $\frac{1-2ac}{2c+abc+1}$ (d) $\frac{1+2ac}{2c-abc-1}$
- 2. If $\log_5 120 + (x-3) 2 \log_5 (1-5^{x-3})$ $= -\log_5 (0.2 - 5^{x-4})$, then x is
 - (a) 1

(c) 3

- 3. If $x_n > x_{n-1} > ... > x_2 > x_1 > 1$, then the value of

 $\log_{x_1} \log_{x_2} \log_{x_3} \dots \log_{x_n} x_n^{x_{n-1}}$, is

(a) 0

(c) 2

- (d) undefined
- 4. The number $\log_{20} 3$ lies in
 - (a) $\left(\frac{1}{4}, \frac{1}{3}\right)$
- (b) $\left(\frac{1}{3}, \frac{1}{2}\right)$
- (c) $\left(\frac{1}{2}, \frac{3}{4}\right)$
- (d) $\left(\frac{3}{4}, \frac{4}{5}\right)$
- 5. The number of real values of the parameter λ for which $(\log_{16} x)^2 - \log_{16} x + \log_{16} \lambda = 0$ with real coefficients will have exactly one solution is
 - (a) 1

(b) 2

(c) 3

- (d) 4
- 6. If $y = 2^{\log_x 4}$, then x is equal to
 - (a) \sqrt{y}

(c) y^2

- (d) y^3
- 7. If $\log_2 x + \log_2 y \ge 6$, then the least value of x + y is

(b) 8

(c) 16

(d) 32

- 8. A rational number which is 50 times its own logarithm to the base 10 is
 - (a) 1

(b) 10

(c) 100

- (d) 1000
- 9. If $x = \log_5 (1000)$ and $y = \log_7 (2058)$, then
 - (a) x > y

(b) x < y

(c) x = y

- (d) none of these
- 10. If $f(x) = \ln\left(\frac{1+x}{1-x}\right)$, then
 - (a) $f(x_1) \cdot f(x_2) = f(x_1 + x_2)$
 - (b) f(x+2) 2f(x+1) + f(x) = 0
 - (c) $f(x) + f(x+1) = f(x^2 + x)$
 - (d) $f(x_1) + f(x_2) = f\left(\frac{x_1 + x_2}{1 + x_1 x_2}\right)$
- 11. If $\log_{10} 2$, $\log_{10} (2^x + 1)$, $\log_{10} (2^x + 3)$ are in AP, then
 - (a) x = 0

- (c) $x = \log_{10} 2$
- (d) $x = \frac{1}{2} \log_2 5$
- 12. If $A = \log_2 \log_2 \log_4 256 + 2 \log_{\sqrt{2}} 2$, then A is equal to
 - (a) 2 (c) 5

- 13. $7 \log \left(\frac{16}{15}\right) + 5 \log \left(\frac{25}{24}\right) + 3 \log \left(\frac{81}{80}\right)$ is equal to

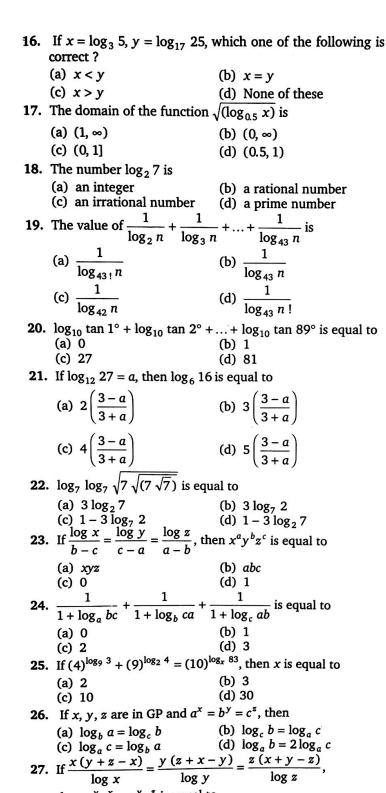
(c) log 2

- (d) log 3
- 14. For $y = \log_a x$ to be defined 'a' must be
 - (a) any positive real number
 - (b) any number
 - (c) ≥ e
 - (d) any positive real number $\neq 1$
- 15. If $\log_{10} 3 = 0.477$, the number of digit in 3^{40} is
 - (a) 18

(b) 19

(c) 20

(d) 21



then $x^y y^x = z^y y^z$ is equal to

(a) $z^x x^z$

(c) $x^y y^z$

is equal to

(a) 1

(c) 3

(b) x^2y^x

(d) $x^x y^y$

(d) 4

28. If $\log_3 2$, $\log_3 (2^x - 5)$, $\log_3 (2^x - 7/2)$ are in AP, then x

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29. If y = a^{\frac{1}{1 - \log_a x}} and z = a^{\frac{1}{1 - \log_a y}}, then x is equal to
  30. If \log_{\sqrt{8}} b = 3\frac{1}{3}, then b is equal to
                                               (b) 8
        (a) 2
        (c) 32
                                              (d) 64
  31. If \log_{0.3}(x-1) < \log_{0.09}(x-1), then x lies in the interval
        (a) (-∞, 1)
                                              (b) (1, 2)
                                             (d) none of these
        (c) (2, ∞)
  32. The value of 3^{\log_4 5} - 5^{\log_4 3} is
        (a) 0
                                             (b) 1
                                             (d) none of these
        (c) 2
 33. If x^{18} = y^{21} = z^{28}, then 3, 3 \log_y x, 3 \log_z y, 7 \log_x z are
       (a) AP
                                             (b) GP
       (c) HP
                                             (d) AGP
 34. If \frac{1}{\log_3 \pi} + \frac{1}{\log_4 \pi} > x, then x be
                                            (d) none of these
35. If \ln\left(\frac{a+b}{3}\right) = \left(\frac{\ln a + \ln b}{2}\right), then \frac{a}{b} + \frac{b}{a} is equal to
       (c) 5
 36. If \log_3 \{5 + 4 \log_3 (x - 1)\} = 2, then x is equal to
                                           (d) \log_2 16
 37. If 2x^{\log_4 3} + 3^{\log_4 x} = 27, then x is equal to
       (a) 2
       (c) 8
38. The interval of x in which the inequality
                     5^{\frac{1}{4}(\log_5^2 x)} \ge 5x^{\frac{1}{5}(\log_5 x)}
      (a) (0, 5^{-2\sqrt{5}}]
                                          (b) [5^{2\sqrt{5}}, \infty)
      (c) both (a) and (b)
                                          (d) none of these
39. The solution set of the equation
      \log_x 2 \log_{2x} 2 = \log_{4x} 2 is
      (a) \{2^{-\sqrt{2}}, 2^{\sqrt{2}}\}
                                          (b) {1/2, 2}
      (c) \{1/4, 2^2\}
                                          (d) none of these
40. The least value of the expression 2 \log_{10} x - \log_x 0.01 is
                                          (b) 4
                                          (d) 8
41. The solution of the equation \log_7 \log_5(\sqrt{x+5} + \sqrt{x}) = 0 is
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(a) 1 (c) 4

(d) 5 **42.** The number of solutions of $\log_4 (x-1) = \log_2(x-3)$ is (a) 3 (b) 1 (c) 2

(d) 0

• Objective Questions Type II [One or more than one correct answer(s)]

In each of the questions below four choices of which one or more than one are correct. You have to select the correct answer(s) accordingly.

- 1. It is given that x = 9 is a solution of the equation $\ln(x^2 + 15a^2) - \ln(a - 2) = \ln\left(\frac{8ax}{a - 2}\right)$, then
 - (a) a = 3
 - (b) $a = \frac{3}{5}$
 - (c) other solution is x = 3/5
 - (d) other solution is x = 15
- **2.** The value of $\log \{ \log_b a \cdot \log_c b \cdot \log_d c \cdot \log_a d \}$ is

(b) log abcd

(c) log 1

- 3. If $\frac{\log_2 x}{4} = \frac{\log_2 y}{6} = \frac{\log_2 z}{3k}$ and $x^3 y^2 z = 1$, then k is equal

 - (a) -8

(c) 0

- (d) $\log_2\left(\frac{1}{256}\right)$
- **4.** If $\frac{\log a}{(b-c)} = \frac{\log b}{(c-a)} = \frac{\log c}{(a-b)}$, then $a^{b+c} \cdot b^{c+a} \cdot c^{a+b}$ is
 - equal to
 - (a) 0
- (c) a + b + c
- (d) $\log_b a \cdot \log_c b \cdot \log_a c$
- 5. The expression

$$5^{\log_{1/5}(1/2)} + \log_{\sqrt{2}}\left(\frac{4}{\sqrt{7} + \sqrt{3}}\right) + \log_{1/2}\left(\frac{1}{10 + 2\sqrt{21}}\right)$$

simplifies to

(a) 6

- (c) $\sqrt{6\sqrt{6\sqrt{6\sqrt{6...\infty}}}}$
- (d) $3^{\log_{1/3}\left(\frac{1}{6}\right)}$
- If $\log_a x = \alpha$, $\log_b x = \beta$, $\log_c x = \gamma$ and $\log_d x = \delta$, $x \neq 1$ and $a, b, c, d \neq 0, > 1$, then $\log_{abcd} x$
 - (a) $\leq \frac{\alpha + \beta + \gamma + \delta}{16}$ (b) $\geq \frac{\alpha + \beta + \gamma + \delta}{16}$
 - (c) $\frac{1}{\alpha^{-1} + \beta^{-1} + \gamma^{-1} + \delta^{-1}}$ (d) $\frac{1}{\alpha \beta \gamma \delta}$

- 7. $\log_p \log_p \sqrt[p]{p} \sqrt[p]{p} \dots \sqrt[p]{p}$, p > 0 and $p \neq 1$, is equal to
 - (a) n

(b) -n

(c) $\frac{1}{n}$

- (d) $\log_{1/p}(p^n)$
- roots equation 8. Sum of the $x + 1 = 2 \log_2 (2^x + 3) - 2 \log_4 (1980 - 2^{-x})$ is
 - (a) $\log_{11} 2$

- (b) $\log_2 11$
- (c) $\log_{11}(0.5)$
- (d) $\log_{0.5} \left(\frac{1}{11} \right)$
- 9. If $a^x = b$, $b^y = c$, $c^z = a$, $x = \log_b a^{k_1}$, $y = \log_c b^{k_2}$, $z = \log_a c^{k_3}$, then $k_1 k_2 k_3$ is equal to
 - (a) 1

(b) *abc*

(c) (xyz)

- (d) 0
- 10. If $\frac{\ln a}{b-c} = \frac{\ln b}{c-a} = \frac{\ln c}{a-b}$, a, b, c > 0, then
 - (a) $a^{b+c} \cdot b^{c+a} \cdot c^{a+b} = 1$
 - (b) $a^{b+c} + b^{c+a} + c^{a+b} > 3$
 - (c) $a^{b+c} \cdot b^{c+a} \cdot c^{a+b} = 3$
 - (d) $a^{b+c} + b^{c+a} + c^{a+b} > 3(3)^{1/3}$
- 11. If $\frac{\ln a}{y-z} = \frac{\ln b}{z-x} = \frac{\ln c}{x-y}$, then

 - (b) $a^{y^2 + yz + z^2} \cdot b^{z^2 + zx + x^2} \cdot c^{x^2 + xy + y^2} = 1$
 - (c) $a^{y+z} \cdot b^{z+x} \cdot c^{x+y} = 1$
 - (d) abc = 1
- 12. The solution of the equation $3^{\log_a x} + 3x^{\log_a 3} = 2 \text{ is given by}$
 - (a) $a^{\log_3 a}$
 - (b) $(2/a)^{\log_3 2}$
 - (c) $a^{-\log_3 2}$
 - (d) $2^{-\log_3 a}$

● Linked-Comprehension Type

In these questions, a passage (paragraph) has been given followed by questions based on each of the passages. You have to answer the questions based on the passage given.

PASSAGE 1

An equation of the form

$$2m\log_a f(x) = \log_a g(x), \, a > 0, \, a \neq 1, \, m \in \mathbb{N}$$

$$\begin{cases} f(x) > 0 \\ f^{2m}(x) = g(x) \end{cases}$$
 is equivalent to the system

On the basis of above information, answer the following questions:

1. The number of solutions of $2\log_e 2x = \log_e (7x - 2 - 2x^2)$

is

(a) 1 (c) 3 (b) 2

(d) infinite

2. The number of solutions of $\ln 2x = 2\ln(4x - 15)$ is

(b) 1

(c) 2

(d) infinite

of solutions 3. The number $\log(3x^2 + x - 2) = 3\log(3x - 2)$ is

(a) 1

(b) 2

(c) 3

(d) 0

4. Solution set of the equation

 $\log_{(x^3+6)}(x^2-1) = \log_{(2x^2+5x)}(x^2-1)$ is

(a) $\{-2\}$

(c) {3}

(d) $\{-2, 1, 3\}$

equation the 5. Solution set of $\log(x-9) + 2\log\sqrt{(2x-1)} = 2$ is

(a) $\{\phi\}$

(b) {1}

(c) {2}

(d) {13}

PASSAGE 2

Equations of the form (i) $f(\log_a x) = 0$, a > 0, $a \ne 1$ and (ii) $g(\log_x A) = 0$, A > 0, then Eq. (i) is equivalent to f(t) = 0, where $t = \log_a x$.

If $t_1, t_2, t_3, \dots, t_k$ are the roots of f(t) = 0, then $\log_a x = t_1, \log_a x = t_2, \dots, \log_a x = t_k$ and Eq. (ii) is equivalent to f(y) = 0, where $y = \log_x A$. If $y_1, y_2, y_3, \ldots, y_k$ are the roots of f(y) = 0, then $\log_x A = y_1, \log_x A = y_2, \ldots, \log_x A = y_k$.

On the basis of above information, answer the following questions:

- equation solutions of number $\frac{1 - 2(\log x^2)^2}{\log x - 2(\log x)^2} = 1 \text{ is}$
 - (a) 0

(b) 1

(c) 2

- (d) infinite
- 2. The number of solutions of the equation $\log_x^3 10 - 6\log_x^2 10 + 11\log_x 10 - 6 = 0$ is

(a) 0

3. The solution set of $(\log_5 x)^2 + \log_5 x + 1 = \frac{7}{\log_5 x - 1}$

contains

- (a) (1, 3)
- (b) {1}

- (c) {25}
- (d) {1, 25} satisfying equation the

4. The set of all x $x^{\log_3 x^2 + (\log_3 x)^2 - 10} = \frac{1}{x^2}$ is

- (a) {1, 9}
- (b) $\{9, \frac{1}{81}\}$
- (c) $\{1, 4, \frac{1}{81}\}$
- (d) $\{1, 9, \frac{1}{81}\}$
- 5. If $\frac{(\ln x)^2 3 \ln x + 3}{\ln x 1} < 1$, then x belongs to
 - (a) (0, e)

- (b) (1, e)
- (c) (1, 2e)
- (d) (0, 3e)

Matrix-Match Type _

Given below are Matching Type Questions, with two columns (each having some items) each . Each item of Column I has to be matched with the items of Column II, by encircling the correct match(es).

NOTE An item of Column I can be matched with more than one items of Column II. All the items of Column II have to be matched.

1. Observe the following columns:

	Column I	Column II					
(A)	If $\log_{34} 5$ lies in the interval (a, b) , then	(P)	[10a+10b]=8, where [.] denotes the greatest integer function				
(B)	If $\log_{300} 4$ lies in the interval (a, b) , then	(Q)	(10a + 10b) = 5 where (.) denotes the least integer function				
(C)	If $\log_{400} 3$ lies in the interval (a, b) , then	(R)	[6b - 3a] = 2 where [.] denotes the greatest integer function				
		(S)	[10a + 10b] = 3 where [.] denotes the greatest integer function				
		(T)	(6b - 3a) = 1 where (.) denotes the least integer function				

(A) (P) (Q) (R) (S) (T)

(B) (P) (Q) (R) (S) (T)

(C) (P) (Q) (R) (S) (T)

2. Observe the following columns:

	Column I		Column II
(A)	The solution set of $\log_{100} x + y = \frac{1}{2}$,	(P)	{√2, 2}
	$\log_{10} y - \log_{10} x = \log_{100} 4 \text{ is}$		
(B)	The solution set of $4\log_2^2 x + 1 = 2\log_2 y$ and $\log_2 x^2 \ge \log_2 y$	(Q)	{1,1}
(C)		(R)	{-10,20}
,		(S)	{4, 2}
	*	(T)	$\left\{\frac{10}{3},\frac{20}{3}\right\}$
(A)	P Q R S T (B) P Q R S T)	(C) P Q B S T

Objective Questions Type I [Only one correct answer]

1.	(a)	2.	(a)	3.	(b)	4.	(b)	5.	(a)	6.	(c)	7.	·(c)	8.	(c)	9.	(a)	10.	(d)
11.	(d)	12.	(c)	13.	(c)	14.	(d)	15.	(c)	16.	(c)	17.	(c)	18.	(c)	19.	(a)	20.	(a)
21.	(c)	22.	(c)	23.	(d)	24.	(b)	25.	(c)	26.	(a)	27.	(a)	28.	(c)	29.	(c)	30.	(c)
31.	(b)	32.	(a)	33.	(a)	34.	(a)	35.	(d)	36.	(b)	37.	(d)	38.	(c)	39.	(a)	40.	(b)
41	(c)	42.	(h)																

Objective Questions Type II [One or more than one correct answer(s)]

4. (b, d) **5.** (a, c, d) 3. (a, d) 1. (a, d) **2.** (a, c) **10.** (a, b) **8.** (b, d) **9.** (a, c) 7. (b, d) **6.** (a, c) **12.** (c, d) **11.** (a, b, c, d)

Linked-Comprehension Type

Passage 2 1. (c) 2. (d) 3. (c) 4. (d) 5. (a) Passage 1 1. (b) 2. (b) 3. (d) 4. (c) 5. (d)

61.	The values of x whice $\sqrt{(5x^2 - 8x + 3)} - \sqrt{(5x^2 - 8x + 3)}$	h satisfy the equation $(5x^2 - 9x + 4)$
	$=\sqrt{(2x^2-2x)}-\sqrt{(2x^2-2x)}$	$2x^2 - 3x + 1$ are
	(a) 3	(b) 2
	(c) 1	(d) 0
62.	The number of number equation $x^2 - xy +$	per-pairs (x, y) which $y^2 = 4(x + y - 4)$ is
	(a) 1	(b) 2
	(c) 4	(d) none of t
63.	The solution set of t	the equation $\log_x 2$ lo

- hese $g_{2x} 2 = \log_{4x} 2$
 - (a) $\{2^{-\sqrt{2}}, 2^{\sqrt{2}}\}$
- (b) $\left\{\frac{1}{2}, 2\right\}$
- (c) $\left\{\frac{1}{4}, 2^2\right\}$
- (d) none of these

will satisfy the

- **64.** For any real x the expression $2(k-x)[x+\sqrt{x^2+k^2}]$ can not exceed
 - (a) k^2

(c) $3k^2$

- (d) none of these
- 65. The solution of $\left| \frac{x}{x-1} \right| + |x| = \frac{x^2}{|x-1|}$ is
 - (a) $x \ge 0$

- (c) $x \in (1, \infty)$
- (d) none of these
- 66. The number of positive integral solutions of $\frac{x^2 (3x-4)^3 (x-2)^4}{(x-5)^5 (2x-7)^6} \le 0 \text{ is}$
 - (a) four

(b) three

(c) two

- (d) only one
- 67. The number of real solutions of the equation $= -3 + x - x^2$ is
 - (a) none

(c) two

- (d) more than two
- **68.** The equation $|x+1|^{\log_{(x+1)}(3+2x-x^2)} = (x-3)|x|$ has
 - (a) unique solution
- (b) two solutions
- (c) no solution
- (d) more than two solutions
- **69.** If xy = 2(x + y), $x \le y$ and $x, y \in N$, the number of solutions of the equation
 - (a) two
 - (b) three
 - (c) no solution
 - (d) infinitely many solutions

70. The number of real solutions of the system of equations
$$x = \frac{2z^2}{1+z^2}, y = \frac{2x^2}{1+x^2}, z = \frac{2y^2}{1+y^2} \text{ is}$$

(a) 1

- 71. The number of negative integral solutions of $x^2 \cdot 2^{x+1} + 2^{|x-3|+2} = x^2 \cdot 2^{|x-3|+4} + 2^{x-1}$ is
 - (a) none

(b) only one

(c) two

(d) four

72. If a be a positive integer, the number of values of a satisfying

$$\int_0^{\pi/2} \left[a^2 \left(\frac{\cos 3x}{4} + \frac{3}{4} \cos x \right) + a \sin x - 20 \cos x \right] dx$$

$$\leq -\frac{a^2}{3} \text{ is}$$

- (a) only one

(c) three

- (d) four
- 73. For the equation $|x^2 2x 3| = b$ which statement or statements are true
 - (a) for b < 0 there are no solutions
 - (b) for b = 0 there are three solutions
 - (c) for 0 < b < 1 there are four solutions
 - (d) for b = 1 there are two solutions
- 74. If y = 2[x] + 3 = 3[x 2] + 5, then [x + y] is ([x] denotes the integral part of x)
 - (a) 10

(b) 15

(c) 12

- (d) none of these
- 75. The roots of the equation $(3-x)^4 + (2-x)^4 = (5-2x)^4$
 - (a) all real
 - (b) all imaginary
 - (c) two real and two imaginary
 - (d) none of the above
- ordered of **76.** The number $(x, y, z, w), (x, y, z, w \in [0, 10])$ which satisfies the inequality $2^{\sin^2 x} 3^{\cos^2 y} 4^{\sin^2 z} 5^{\cos^2 w} \ge 120$ is
 - (a) 0

(b) 144

(c) 81

- (d) infinite
- function be a 77. Let F(x) $F(x) = x - [x], 0 \neq x \in R$, where [x] is the greatest integer less than or equal to x. Then the number of solutions of F(x) + F(1/x) = 1 is/are
 - (a) 0

(b) infinite (d) 2

- (c) 1
- 78. The largest interval in which $x^{12} x^9 + x^4 x + 1 > 0$ is
 - (a) [0, ∞)
- (b) $(-\infty, 0]$
- (c) (-∞,∞)
- (d) none of these
- 79. The system of equation |x-1| + 3y = 4, |x-y-1| = 2
 - (a) no solution
 - (b) a unique solution
 - (c) two solutions
 - (d) more than two solutions
- **80.** If $5\{x\} = x + [x]$ and $[x] \{x\} = \frac{1}{2}$, where $\{x\}$ and [x] are

fractional and integral part of x, then x is

(a) 1/2

(c) 5/2

- (d) 7/2
- 81. If c > 0 and the equation $3ax^2 + 4bx + c = 0$ has no real root, then
 - (a) 2a + c > b
- (b) a + 2c > b
- (c) 3a + c > 4b
- (d) a + 3c < b

Answers:-

- **62.** (a) **61**. (c) **71.** (a)
 - **72.** (d)
- **63.** (a) **73.** (a)
- **64.** (b) 74. (b)
- **65.** (c) 75. (c)
- **66**. (b)
- 76. (b)
- 67. (a) 77. (b)
- 68. (c) (c)
- (a)
- (a) 80. (b)

81. (c)